TWISTORS

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Introduction

This project started out when Nat Friedman asked Robert Krawczyk to design a computer generated version of the aluminum triple twist Mobius band in Figure 1(a). The first version is shown in Figure 1(b). The digital version has all of the same geometric aspects of the aluminum sculpture; the corner connections and the variable change in speed of the twist itself. After some discussion, there followed a whole series of sculptures with twists which we refer to as *twistors* for short. Some, but not all, twistors are one-sided. Another triple twist Mobius is shown in Figure 2. Here the twist has no straight parts on sides.

(a) Aluminum, 9 x 9 x 2 inches. *(b) Computer Generated*

Figure 1. Physical and computer generated versions of Triple twist Mobius band 1.

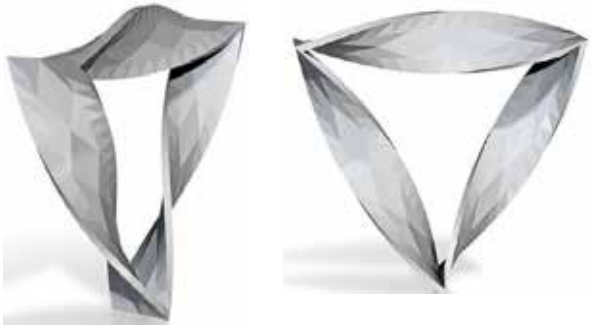

Figure 2.Two views of Triple twist Mobius band 2.

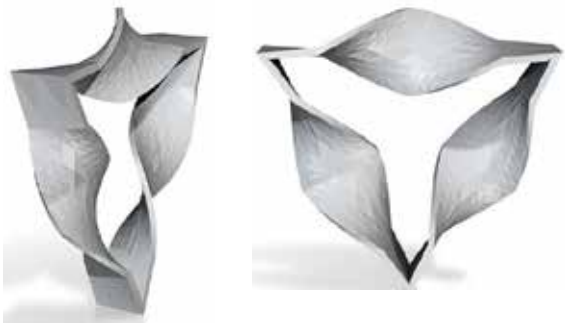

Figure 3. Two views of Triple twist Mobius band 3.

A third version is shown in Figure 3 , where the twist is enlarged and more pronounced.
From a mathematical viewpoint, a twist is just a twist. From a sculptural viewpoint, there are variously shaped twists, such as long and slow, tight and fast, etc. and we will consider various twist shapes below.

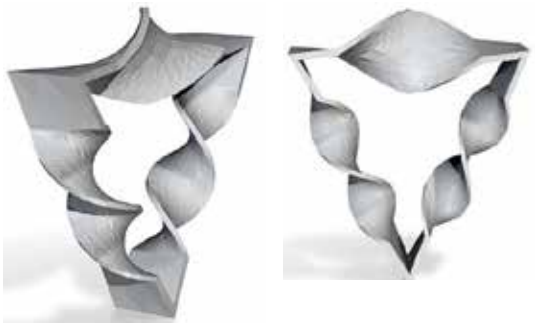

Figure 4. Two views of Five-twist Mobius band 1.

Figure 5. Two views of Seven-twist Mobius Band 1.

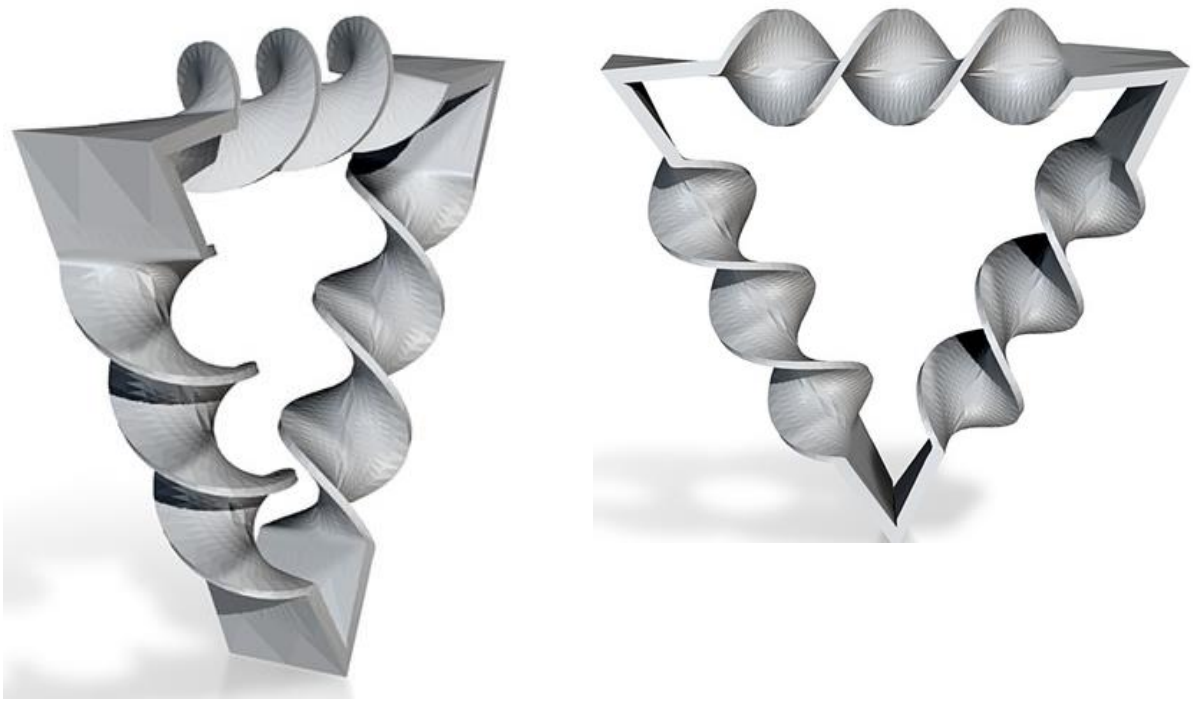

Figure 6. Two views of Nine-twist Mobius Band 1.

The next twistor is a five-twist Mobius band, as shown in Figure 4. We stayed with the triangular arrangement with twists of two sizes for variety, rather than a pentagon with five equal size twists.

A seven-twist Mobius band is shown in Figure 5 with twists of two sizes. Also note that the top twists are even tighter than those in Figure 4.

Lastly, a nine-twist Mobius band is shown in Figure 6. Here the twists are the same size.

Divided Bands

It is well known that a divided Mobius band remains as one single two-sided band. The twistor in Figure 1 is shown divided in Figure 7. The shape of the dividing space matches the shape of the sculpture, hence the space is a triple twist Mobius space. In Figure7.(c) the ends are joined for support as, for example, in the case of a large outdoor sculpture.

(a) *(b)* *(c)*

Figure 7. Three views of Triple Twist Mobius Band 1 divided.

In Figure 8 are views of the divided nine-twist Mobius Band 1, where a more supportive join is shown in the ends in Figure 8(c). The space is a nine-twist Mobius space.

(a) *(b)* *(c)*

Figure 8. Three views of Nine-twist Mobius Band 1 divided.

Even Number of Twists

We have also experimented with even number of twists. For even number of twists, the twistors are two-sided and the divided twistor consists of two pieces.

A twistor with six twists (six twistor) is shown in Figure 9.

A divided twistor with six twists is shown with just the ends joined in Figure 10.

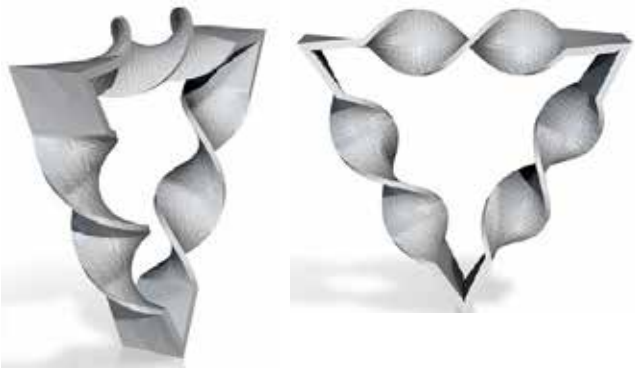

Figure 9. Two views of Six Twistor 1.

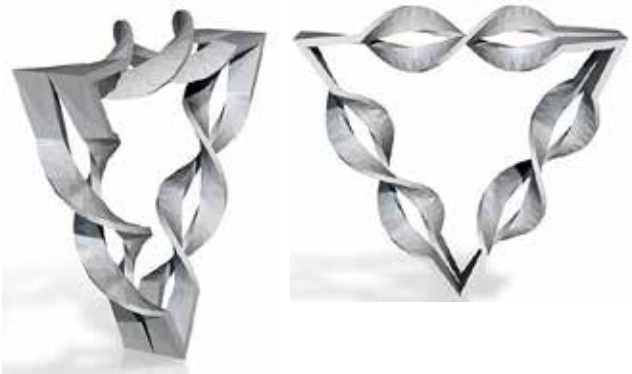

Figure 10. Two views of Six Twistor 1 divided.

An alternate six twist is shown in Figure 11, where quarter twists are on each side of a half twist.

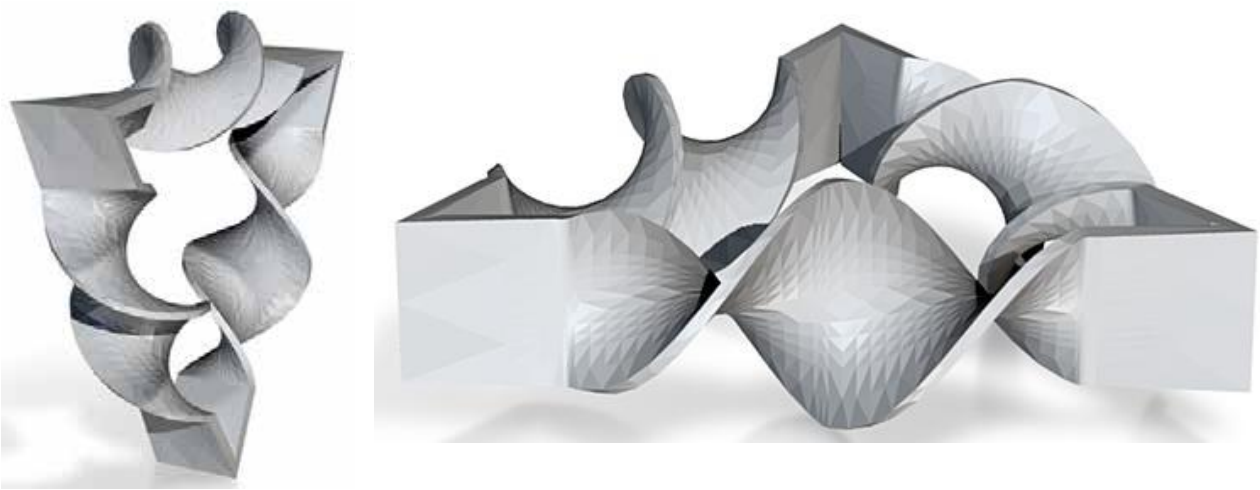

Figure 11. Two views of Alternate Six Twistor.

Mirrored Twistors

The twists in Figure 1(a) are considered right-handed and the twists in Figure 1(b,c) are left-handed. One is the mirror image of the other. When the two mirror images are combined, we refer to this as a *mirrored twistor*. The mirrored twistor corresponding to the twistor in Figure 3 is shown in Figure 12. The mirrored twistor has self-intersections of the component twistors.

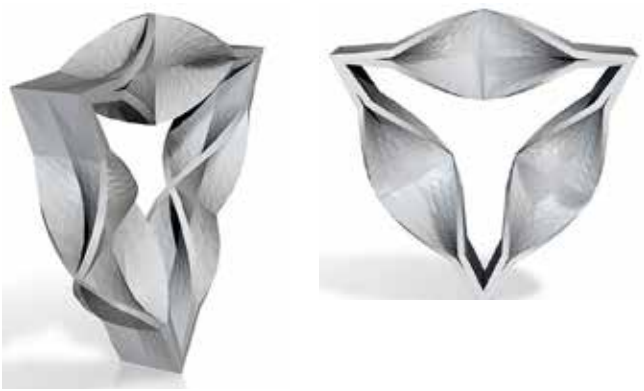

Figure 12. Two views of Mirrored Triple Twist Mobius 3.

Figure 13. Two views of Mirrored Seven twist Mobius Band 1.

The mirrored version of the seven twist Mobius band in Figure 5 is shown in Figure 13. The divided mirrored triple twist Mobius band 1 is shown in Figure 14.

Figure 14. Two views of Divided mirrored triple twist Mobius Band 1.

Rotation Combination

The sculpture in Figure 15 is obtained by rotating the twistor in Figure 14 a quarter turn and then combining the rotation with itself. A top view is also shown.

(a) Side View *(b) Top View*

Figure 15. Two views of rotation combination of divided mirrored triple twist Mobius Band 1.

The top view in Figure 15 shows that the sculpture can again be rotated a quarter turn and then combined with itself resulting in the sculpture in Figure 16.

(a) Side View *(b) Top View*

Figure 16. Two views of double rotation combination of divided mirrored triple twist Mobius Band 1.

Three- way Division

If a Mobius band is divided into thirds, then the middle third is a separate Mobius band and the two outer thirds form one two-sided band as in the case of cutting a Mobius band in half. This is shown in Figure 17 for the twistor in Figure 1. The ends are joined for support since the middle band is separated from the outer joined bands.

Figure 17. Two views of Triple twist Mobius band 1 divided in three.

Figure 18. Two views of Nine twist Mobius Band 1 divided in three.

The nine twistor 1 divided in three is shown in Figure 18. The corresponding mirrored twistor is shown in Figure 19.

Figure 19. Two views of Nine twist Mobiuis Band 1 mirrored divided in three.

Rotation in the Paper Plane

An image can be rotated in the paper plane and combined with the image as shown in Figure 20, where the image is six twist 1 mirrored.

Figure 20. Two views of Six twist 1 mirrored rotated in plane and combined.

In Figure 21 six twist 1 mirrored is rotated a quarter turn left and right and the two rotations are combined.

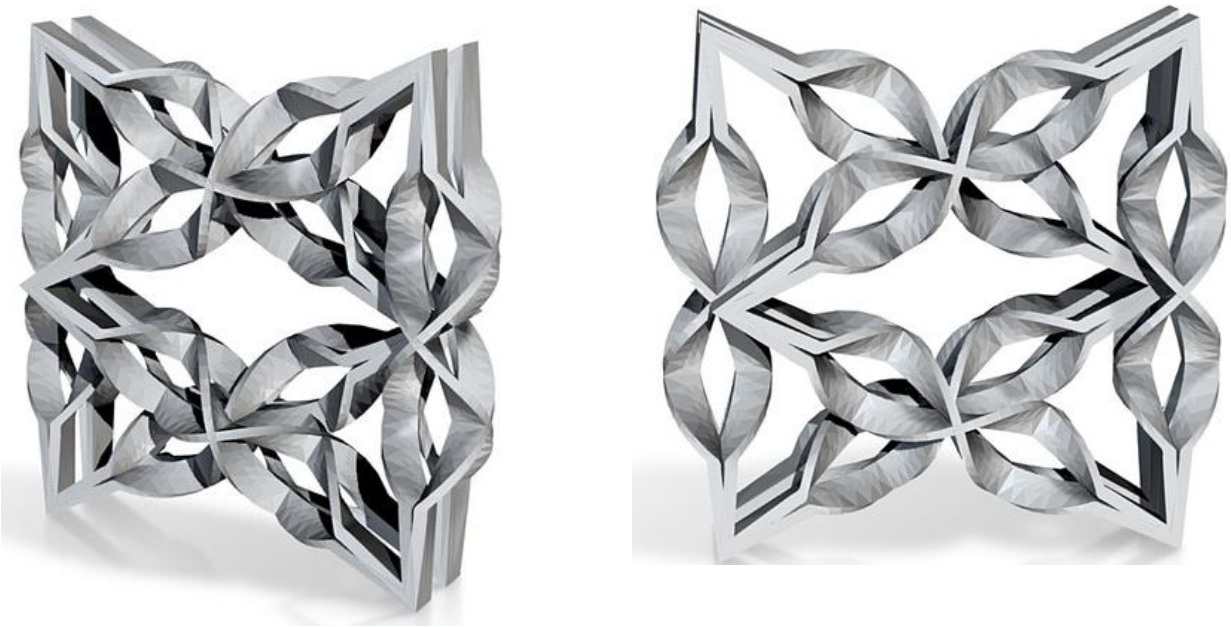

Figure 21. Two views of Six twist 1 mirrored with right and left quarter turns combined.

Vertical Mirror Image In the Paper Plane

We consider starting with divided six twistor 1 in Figure 22. A quarter turn rotation of Six twistor 1 divided is now combined with itself, as in Figure 23.

Figure 22. Two views of Six twistor 1 divided.

(a) Side View *(b) Top View*

Figure 23. Two views of Quarter turn rotation of Six twistor 1 divided combined with itself.

The twistor in Figure 23 is now combined with an eighth turn rotation of itself and then mirror imaged to obtain the twistor in Figure 24. The twistor in Figure 24 is now vertically mirror imaged in the paper plane to obtain the sculpture in Figure 25.

Figure 24. Two views of Rotation combination and mirror image.

Figure 25. Vertical mirror image in the paper plane.